USN | | | | | | | | | |

MATDIP301

Third Semester B.E. Degree Examination, December 2010
Advanced Mathematics – I

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions.***

1 a. Find the n^{th} derivative of $\log(ax + b)$. **(06 Marks)**

b. Find the n^{th} derivative of $\frac{x}{(1+3x+2x^2)}$. **(07 Marks)**

c. If $x = \sin t$ and $y = \text{cons } mt$, prove that $(1 - x^2)\, y_{n+2} - (2n + 1)\, xy_{n+1} + (m^2 - n^2)\, y_n = 0$. **(07 Marks)**

2 a. Show that the following pair of curves intersect each other orthogonally.
$r = a(1 + \sin\theta)$ and $r = a(1 - \sin\theta)$. **(06 Marks)**

b. Find the pedal equation of the curve $\frac{2a}{r} = 1 + \cos\theta$. **(07 Marks)**

c. Find the first five terms of the Maclaurin series of $f(x) = \log \sec x$. **(07 Marks)**

3 a. If $u = e^{ax - by} \sin(ax + by)$, show that $b\frac{\partial u}{\partial x} - \frac{\partial u}{\partial y} = 2abu$. **(06 Marks)**

b. If $u = \sqrt{x^2 + y^2}$ and $x^3 + y^3 + 3axy = 5a^2$, find $\frac{du}{dx}$ when $x = y = a$. **(07 Marks)**

c. If $z = f(x, y)$, where $x = r\cos\theta$ and $y = r\sin\theta$, show that,
$\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 = \left(\frac{\partial z}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial z}{\partial \theta}\right)^2$ **(07 Marks)**

4 a. Obtain the reduction formula for $\int \cos^n x\,dx$, where n is a positive integer. **(06 Marks)**

b. Show that $\int_0^{\pi} \frac{\sqrt{1-\cos\theta}}{1+\cos\theta} \sin^2\theta \, d\theta = \frac{8\sqrt{2}}{3}$. **(07 Marks)**

c. Evaluate $\int_0^a \int_0^{\sqrt{a^2-x^2}} x^2 y \, dy\, dx$. **(07 Marks)**

5 a. Prove that $\Gamma\left(\tfrac{1}{2}\right) = \sqrt{\pi}$. **(06 Marks)**

b. Show that $\int_0^{\pi/2} \sqrt{\sin\theta}\, d\theta \times \int_0^{\pi/2} \frac{d\theta}{\sqrt{\sin\theta}} = \pi$. **(07 Marks)**

c. Prove that $\beta(m,n) = \frac{\Gamma(m)\,\Gamma(n)}{\Gamma(m+n)}$. **(07 Marks)**

6 a. Solve $(e^4 + 1) \cos x\, dx + e^4 \sin x\, dy = 0$. **(06 Marks)**

b. Solve $\left(x \tan \frac{y}{x} - y \sec^2 \frac{y}{x}\right) dx + x \sec^2\left(\frac{y}{x}\right) dy = 0$. **(07 Marks)**

c. Solve $(x + \tan y)\, dy = \sin 2y\, dx$. **(07 Marks)**

7 a. Solve $\frac{d^2y}{dx^2}+3\frac{dy}{dx}+2y=e^{-2x}$. (06 Marks)

b. Solve $\frac{d^2y}{dx^2}+2\frac{dy}{dx}-5y=\cos 3x$. (07 Marks)

c. Solve $(D^2-5D+1)y=1+x^2$. (07 Marks)

8 a. Prove that $(1+\cos\theta+i\sin\theta)^n+(1+\cos\theta-i\sin\theta)^n=2^{n+1}\cos^n\left(\frac{\theta}{2}\right)\cos\left(\frac{n\theta}{2}\right)$ (06 Marks)

b. Use Demoivre's theorem and solve the equation $x^4-x^3+x^2+1=0$. (07 Marks)

c. Expand $\cos^8\theta$ in a series of cosine of multiples of θ. (07 Marks)

* * * * *